The 3-3-1-1 Model

Nguyen Thi Thuy
Yonsei University

High1 Resort, Feb. 9 - 15, 2014

Outline

1. Motivation
2. Particle content
3. Scalar sector
4. Gauge sector
5. Fermion sector
6. Summary

1. Motivation

▷ There exists a simple extension of the SM gauge group to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, the so called 3-3-1 models. There are three main versions: the minimal model, the version with right-handed (RH) neutrinos and the version with neutral fermions.

1. Motivation

- There exists a simple extension of the SM gauge group to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, the so called 3-3-1 models. There are three main versions: the minimal model, the version with right-handed (RH) neutrinos and the version with neutral fermions.
- In 331 model, the lepton number is considered as global symmetry.
- In 3311 model, the lepton number is considered as local symmetry.

Lepton number

- ▷ In the SM, the lepton number operator commutes with the generators of the unitary group $SU(2)_L$.

Lepton number

- In the SM, the lepton number operator commutes with the generators of the unitary group $SU(2)_L$.
- In the 3-3-1 model the lepton number of 3 components in a triplet are different, so the lepton number operator does not commute with the generators of the unitary group $SU(3)_L$.
  For example $\psi_{aL} = (\nu_{aL}, e_{aL}, (N_{aR})^c)^T$ has the lepton number (1,1,0) then the commutations $[L, T_4], [L, T_5], [L, T_6], [L, T_7] \neq 0$, where T_4, T_5, T_6, T_7 are the generators of $SU(3)_L$ containing new gauge bosons X, Y.

Lepton number

- In the SM, the lepton number operator commutes with the generators of the unitary group $SU(2)_L$.
- In the 3-3-1 model the lepton number of 3 components in a triplet are different, so the lepton number operator does not commute with the generators of the unitary group $SU(3)_L$.
  For example $\psi_{aL} = (\nu_{aL}, e_{aL}, (N_{aR})^c)^T$ has the lepton number (1,1,0) then the commutations $[L, T_4], [L, T_5], [L, T_6], [L, T_7] \neq 0$, where T_4, T_5, T_6, T_7 are the generators of $SU(3)_L$ containing new gauge bosons X, Y.
- In the 3-3-1 model, one constructed lepton number operator as the combination of T_3, T_8, and charged $\mathcal{L}$. One considered $U(1)_{\mathcal{L}}$ as global group.

Lepton number

- ▷ In the SM, the lepton number operator commutes with the generators of the unitary group $SU(2)_L$.
- ▷ In the 3-3-1 model the lepton number of 3 components in a triplet are different, so the lepton number operator does not commute with the generators of the unitary group $SU(3)_L$.
  For example $\psi_{aL} = (\nu_{aL}, e_{aL}, (N_{aR})^c)^T$ has the lepton number (1,1,0) then the commutations $[L, T_4], [L, T_5], [L, T_6], [L, T_7] \neq 0$, where T_4, T_5, T_6, T_7 are the generators of $SU(3)_L$ containing new gauge bosons X, Y.
- ▷ In the 3-3-1 model, one constructed lepton number operator as the combination of T_3, T_8, and charged $\mathcal{L}$. One considered $U(1)_{\mathcal{L}}$ as global group.
- ▷ Since T_3, T_8 are gauged charges of the $SU(3)_L$ symmetry, $L, \mathcal{L}$ should be gauged or local generators.

The 3-3-1-1 model is based on the gauge symmetry

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N.$$

$T_i = \frac{1}{2}\lambda_i (i = 1, 2, 3, ..., 8)$ and X, N are $SU(3)_L$, $U(1)_X$ and $U(1)_N$ charges, respectively.
λ_i are Gell-Mann matrices.

The 3-3-1-1 model is based on the gauge symmetry

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N.$$

$T_i = \frac{1}{2}\lambda_i (i = 1, 2, 3, ..., 8)$ and X, N are $SU(3)_L$, $U(1)_X$ and $U(1)_N$ charges, respectively.
λ_i are Gell-Mann matrices.
The new charge X is connected with the electric charge operator Q through a relation

$$Q = \alpha T_3 + \beta T_8 + XI, \quad \alpha = 1, \beta = -\frac{1}{\sqrt{3}}.$$

The relation between L and $\mathcal{L}$ is obtained

$$L = \alpha' T_3 + \beta' T_8 + \mathcal{L} I, \quad \alpha' = 0, \beta' = \frac{2}{\sqrt{3}}.$$

Why $N = \mathcal{B} - \mathcal{L}$?

Why $N = \mathcal{B} - \mathcal{L}$?

▷ If $N = \mathcal{L}$, the leptonic anomalies are not canceled.

$$[SU(3)_C]^2 U(1)_{\mathcal{L}}, [SU(3)_L]^2 U(1)_{\mathcal{L}}, [U(1)_X]^2 U(1)_{\mathcal{L}}, U(1)_X [U(1)_{\mathcal{L}}]^2, [U(1)_{\mathcal{L}}]^3 \neq 0.$$

Why $N = \mathcal{B} - \mathcal{L}$?

- If $N = \mathcal{L}$, the leptonic anomalies are not canceled.

$$[SU(3)_C]^2 U(1)_{\mathcal{L}}, [SU(3)_L]^2 U(1)_{\mathcal{L}}, [U(1)_X]^2 U(1)_{\mathcal{L}}, U(1)_X [U(1)_{\mathcal{L}}]^2, [U(1)_{\mathcal{L}}]^3 \neq 0.$$

- Define $B = \mathcal{B}I, N = \mathcal{B} - \mathcal{L}$, the anomalies associated with $U(1)_N$ and with the usual $3-3-1$ asymmetry obviously vanish.

2. Particle content

The fermion content of the 3-3-1-1 model which is anomaly free is given as

$$\begin{aligned}
\psi_{aL} &= \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix} \sim (1,3,-1/3,-2/3), \\
\nu_{aR} &\sim (1,1,0,-1), \quad e_{aR} \sim (1,1,-1,-1), \\
Q_{\alpha L} &= \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3,3^*,0,0), \quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3,3,1/3,2/3)\,, \\
u_{aR} &\sim (3,1,2/3,1/3)\,, \quad d_{aR} \sim (3,1,-1/3,1/3)\,, \\
U_R &\sim (3,1,2/3,4/3)\,, \quad D_{\alpha R} \sim (3,1,-1/3,-2/3)\,,
\end{aligned}$$

where the quantum numbers located in the parentheses are defined upon the gauge symmetries $(SU(3)_C,\ SU(3)_L,\ U(1)_X,\ U(1)_N)$, respectively. The family indices are $a = 1,2,3$ and $\alpha = 1,2$.
The N_{aR} are the neutral leptons and U, D_α are the exotic quarks.

To break the gauge symmetry and generate the masses in a correct way, the 3-3-1-1 model needs the following scalar multiplets :

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1, 3, 2/3, 1/3),$$

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1, 3, -1/3, 1/3),$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1, 3, -1/3, -2/3),$$

$$\phi \sim (1, 1, 0, 2),$$

with the VEVs conserving Q and P respectively given by

$$\langle\rho\rangle = \frac{1}{\sqrt{2}}(0, v, 0)^T, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}(u, 0, 0)^T, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}(0, 0, \omega)^T, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda.$$

To break the gauge symmetry and generate the masses in a correct way, the 3-3-1-1 model needs the following scalar multiplets :

$$\begin{aligned}
\rho &= \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1,3,2/3,1/3),\\
\eta &= \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1,3,-1/3,1/3),\\
\chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1,3,-1/3,-2/3),\\
\phi &\sim (1,1,0,2),
\end{aligned}$$

with the VEVs conserving Q and P respectively given by

$$\langle\rho\rangle = \frac{1}{\sqrt{2}}(0,v,0)^T, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}(u,0,0)^T, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}(0,0,\omega)^T, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda.$$

The gauge group $SU(3)_L \otimes U(1)_X \otimes U(1)_N$ is broken:

$$SU(3)_L \otimes U(1)_X \otimes U(1)_N \to U(1)_Q \otimes U(1)_{B-L}.$$

The $\mathcal{L}, \mathcal{B}, N$ charge of model multiplets

Multiplet	ψ_{aL}	ν_{aR}	e_{aR}	Q_{3L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ρ	η	χ	ϕ
$\mathcal{L}$	$\frac{2}{3}$	1	1	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	-1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	-2
$\mathcal{B}$	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0
$N=\mathcal{B}-\mathcal{L}$	$-\frac{2}{3}$	-1	-1	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	2

The lepton number of model particles

L	Particle
0	$N_{aR}, u_a, d_a, \rho_1^+, \rho_2^0, \eta_1^0, \eta_2^-, \chi_3^0$
1	$\nu_{aL}, e_a, \boxed{\bar{U}, D_\alpha}, \rho_3^-, \eta_3^{0*}, \chi_1^0, \chi_2^-$

The R parity of model particles

$P=(-1)^{3(B-L)+2s}$	Particle
+1	$\nu_L, \nu_R, e, u, d, \rho_1, \rho_2, \eta_1, \eta_2, \chi_3, \phi$
-1	$N_R, U, D, \rho_3, \boxed{\eta_3, \chi_1}, \chi_2$

The Lagrangian of the 3-3-1-1 model is given by

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{\text{fermion multiplets}} \bar{F} i\gamma^\mu D_\mu F + \sum_{\text{scalar multiplets}} (D^\mu S)^\dagger (D_\mu S) \\ & -\frac{1}{4} G_{i\mu\nu} G_i^{\mu\nu} - \frac{1}{4} A_{i\mu\nu} A_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} \\ & -V(\rho, \eta, \chi, \phi) + \mathcal{L}_{\text{Yukawa}},\end{aligned}$$

with the covariant derivative

$$D_\mu = \partial_\mu + i g_s T_i G_{i\mu} + i g T_i A_{i\mu} + i g_X X B_\mu + i g_N N C_\mu,$$

and the field strength tensors

$$\begin{aligned}G_{i\mu\nu} &= \partial_\mu G_{i\nu} - \partial_\nu G_{i\mu} - g_s f_{ijk} G_{j\mu} G_{k\nu}, \\ A_{i\mu\nu} &= \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} - g f_{ijk} G_{j\mu} A_{k\nu}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \quad C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.\end{aligned}$$

The Yukawa interactions and scalar potential are obtained as

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= h^e_{ab}\bar{\psi}_{aL}\rho e_{bR} + h^\nu_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^\nu_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + h^U\bar{Q}_{3L}\chi U_R + h^D_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* D_{\beta R} \\
&+h^u_a\bar{Q}_{3L}\eta u_{aR} + h^d_a\bar{Q}_{3L}\rho d_{aR} + h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} + h^u_{\alpha a}\bar{Q}_{\alpha L}\rho^* u_{aR} + H.c,
\end{aligned}
$$

$$
\begin{aligned}
V(\rho,\eta,\chi,\phi) &= \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\chi^\dagger\chi)^2 + \lambda_3(\eta^\dagger\eta)^2 \\
&+\lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) \\
&+\lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) + \lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + (f\epsilon^{mnp}\eta_m\rho_n\chi_p + H.c.) \\
&+\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\eta^\dagger\eta) + \lambda_{12}(\phi^\dagger\phi)(\chi^\dagger\chi).
\end{aligned}
$$

3. Scalar sector

We expand the neutral scalars around their VEVs such as

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}; \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix}; \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix};$$

$$\phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4).$$

3. Scalar sector

We expand the neutral scalars around their VEVs such as

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}; \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix}; \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix};$$

$$\phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4).$$

We assume that f, ω are the same order and $\Lambda \gg \omega \gg u, v$. The physical fields with respective masses can be written as:

For charged scalars,

$$H_4^- = \frac{v\chi_2^- + \omega\rho_3^-}{\sqrt{v^2+\omega^2}}, \quad H_5^- = \frac{v\eta_2^- + u\rho_1^-}{\sqrt{u^2+v^2}},$$

$$m_{H_4}^2 = \left(\frac{1}{2}\lambda_7 - \frac{fu}{\sqrt{2}v\omega}\right)(v^2 + \omega^2), \quad m_{H_5}^2 = \left(\frac{1}{2}\lambda_8 - \frac{f\omega}{\sqrt{2}uv}\right)(u^2 + v^2).$$

$$G_Y^- = \frac{\omega\chi_2^- - v\rho_3^-}{\sqrt{v^2+\omega^2}}, G_W^- = \frac{u\eta_2^- - v\rho_1^-}{\sqrt{u^2+v^2}}.$$

The pseudoscalar A_4 is massless.

$$A = \frac{u^{-1}A_1 + v^{-1}A_2 + \omega^{-1}A_3}{\sqrt{u^{-2}+v^{-2}+\omega^{-2}}}, \quad m_A^2 = -\frac{f}{\sqrt{2}}\frac{u^2v^2+u^2\omega^2+v^2\omega^2}{uv\omega}.$$

$$G_Z = \frac{-uA_1+vA_2}{\sqrt{u^2+v^2}}, \quad G_{Z'} = \frac{-\omega^{-1}(u^{-1}A_1+v^{-1}A_2)+(u^{-2}+v^{-2})A_3}{\sqrt{(u^{-2}+v^{-2}+\omega^{-2})(u^{-2}+v^{-2})}},$$

$$G_X = \frac{\omega\chi_1 - u\eta_3^*}{\sqrt{u^2+\omega^2}}, \quad H' = \frac{u\chi_1^* + \omega\eta_3}{\sqrt{u^2+\omega^2}}, \quad m_{H'}^2 = (\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega})(u^2+\omega^2).$$

For neutral scalars,

$$\begin{aligned}
H &= \frac{uS_1+vS_2}{\sqrt{u^2+v^2}}, \quad m_H^2 = \frac{v^4(4\lambda\lambda_1-\lambda_{10}^2)-u^4(\lambda_{11}^2-4\lambda\lambda_3)-2u^2v^2(\lambda_{10}\lambda_{11}-2\lambda\lambda_5)}{4(u^2+v^2)\lambda}\\
&\quad +\frac{1}{4\sqrt{2}(u^2+v^2)\lambda(\lambda_{12}^2-4\lambda\lambda_2)}(m_0+m_1\frac{f}{\omega}+m_2\frac{f^2}{\omega^2}),\\
H_1 &= \frac{-vS_1+uS_2}{\sqrt{u^2+v^2}}, \quad m_{H_1}^2 = -\frac{f(u^2+v^2)\omega}{\sqrt{2}uv},\\
H_2 &= S_3, \quad m_{H_2}^2 = \frac{(4\lambda\lambda_2-\lambda_{12}^2)\omega^2}{2\lambda},\\
H_3 &\simeq S_4, \quad m_{H_3}^2 = 2\lambda\Lambda^2,
\end{aligned}$$

where

$$\begin{aligned}
m_0 &= \sqrt{2}(v^2(\lambda_{10}\lambda_{12}-2\lambda\lambda_4)+u^2(\lambda_{11}\lambda_{12}-2\lambda\lambda_6))^2,\\
m_1 &= 8uv\lambda(v^2(-\lambda_{10}\lambda_{12}+2\lambda\lambda_4)+u^2(-\lambda_{11}\lambda_{12}+2\lambda\lambda_6)),\\
m_2 &= 8\sqrt{2}u^2v^2\lambda^2.
\end{aligned}$$

H is identified as SM Higgs boson.

4. Gauge sector

The gauge mass Lagrangian is given as

$$\begin{aligned}\mathcal{L}_{gaugemass} &= (0,0,\frac{\omega}{\sqrt{2}})(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu - \frac{2}{3}g_N C_\mu)^2(0,0,\frac{\omega}{\sqrt{2}})^T \\ &+(\frac{u}{\sqrt{2}},0,0)(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(\frac{u}{\sqrt{2}},0,0)^T \\ &+(0,\frac{v}{\sqrt{2}},0)(gA_{a\mu}T_a + \frac{2}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(0,\frac{v}{\sqrt{2}},0)^T \\ &+2(g_N C_\mu \Lambda)^2.\end{aligned}$$

4. Gauge sector

The gauge mass Lagrangian is given as

$$\begin{aligned}\mathcal{L}_{gaugemass} &= (0,0,\frac{\omega}{\sqrt{2}})(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu - \frac{2}{3}g_N C_\mu)^2(0,0,\frac{\omega}{\sqrt{2}})^T \\ &+(\frac{u}{\sqrt{2}},0,0)(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(\frac{u}{\sqrt{2}},0,0)^T \\ &+(0,\frac{v}{\sqrt{2}},0)(gA_{a\mu}T_a + \frac{2}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(0,\frac{v}{\sqrt{2}},0)^T \\ &+2(g_N C_\mu \Lambda)^2.\end{aligned}$$

$$W_\mu^\pm = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}}, Y_\mu^\mp = \frac{A_{6\mu} \mp iA_{7\mu}}{\sqrt{2}}.$$

$$M_W^2 = \frac{1}{4}g^2(u^2+v^2), M_Y^2 = \frac{1}{4}g^2(v^2+\omega^2).$$

$$X_\mu^0 = \frac{A_{4\mu} - iA_{5\mu}}{\sqrt{2}}, M_X^2 = \frac{1}{4}g^2(u^2+\omega^2).$$

Set $t_1 \equiv g_X/g$, $t_2 \equiv g_N/g$.

There is a mixing among $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ components. The mass mixing matrix of $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ contains one exact eigenvalues with the corresponding eigenstates as follows

$$M_\gamma^2 = 0, A_\mu = \frac{\sqrt{3}}{\sqrt{3+4t_1^2}}\left(t_1 A_{3\mu} - \frac{t_1}{\sqrt{3}} A_{8\mu} + B_\mu\right).$$

In the limit $\Lambda \gg \omega$

$$\begin{aligned}
Z^N_\mu &\sim C_\mu, \quad m^2_{Z^N} \simeq 4g^2 t_2^2 \Lambda^2, \\
Z^1_\mu &= \cos\xi Z_\mu - \sin\xi Z'_\mu, \quad Z^2_\mu = \sin\xi Z_\mu + \cos\xi Z'_\mu, \\
m^2_{Z^1} &\simeq \frac{g^2}{8}\Bigg(u^2+\omega^2+\frac{u^2+4v^2+\omega^2}{3-4s^2_W} \\
&- 4\frac{\sqrt{c^4_W u^4 + v^4 - c_{2W}v^2\omega^2 + c^4_W\omega^4 + u^2(-c_{2W}v^2+(-1+2s^4_W)\omega^2)}}{(3-4s^2_W)}\Bigg), \\
m^2_{Z^2} &\simeq \frac{g^2}{8}\Bigg(u^2+\omega^2+\frac{u^2+4v^2+\omega^2}{3-4s^2_W} \\
&+ 4\frac{\sqrt{c^4_W u^4 + v^4 - c_{2W}v^2\omega^2 + c^4_W\omega^4 + u^2(-c_{2W}v^2+(-1+2s^4_W)\omega^2)}}{(3-4s^2_W)}\Bigg),
\end{aligned}$$

where

$$\tan 2\xi = \frac{\sqrt{3-4s^2_W}(c_{2W}u^2 - v^2)}{((-1+2s^4_W)u^2 - c_{2W}v^2 + 2c^4_W\omega^2)},$$

$$Z_\mu = \frac{\sqrt{3+t_1^2}}{\sqrt{3+4t_1^2}}A_{3\mu} + \frac{t_1(\sqrt{3}t_1 A_{8\mu} - 3B_\mu)}{\sqrt{3+t_1^2}\sqrt{3+4t_1^2}}, \quad Z'_\mu = \frac{\sqrt{3}}{\sqrt{3+t_1^2}}A_{8\mu} + \frac{t_1}{\sqrt{3+t_1^2}}B_\mu.$$

If we assume $\omega \gg u, v$, then $\tan 2\xi \to 0$. We get

$$Z^1_\mu \sim Z_\mu, \quad m^2_{Z^1} \simeq \frac{g^2(u^2+v^2)}{4c^2_W},$$
$$Z^2_\mu \sim Z'_\mu, \quad m^2_{Z^2} \simeq \frac{c^2_{2W}u^2+v^2+4c^4_W\omega^2}{(3-4s^2_W)c^2_W}.$$

The gauge boson Z^1_μ is identified as Z_μ in the SM.

5. Fermion sector

The Dirac masses are written in the form $-\bar{f}_L m_f f_R +$ H.c. From the $\mathcal{L}_{\text{Yukawa}}$, we obtain

$$\begin{aligned}
m_U &= -\frac{1}{\sqrt{2}} h^U \omega, \quad [m_D]_{\alpha\beta} = -\frac{1}{\sqrt{2}} h^D_{\alpha\beta}\omega, \\
[m_u]_{\alpha a} &= \frac{1}{\sqrt{2}} h^u_{\alpha a} v, \quad [m_u]_{3a} = -\frac{1}{\sqrt{2}} h^u_a u, \\
[m_d]_{\alpha a} &= -\frac{1}{\sqrt{2}} h^d_{\alpha a} v, \quad [m_d]_{3a} = -\frac{1}{\sqrt{2}} h^d_a v, \\
[m_e]_{ab} &= -\frac{1}{\sqrt{2}} h^e_{ab} v, \quad [m^D_\nu]_{ab} = -\frac{1}{\sqrt{2}} h^\nu_{ab} u.
\end{aligned}$$

5. Fermion sector

The Dirac masses are written in the form $-\bar{f}_L m_f f_R + \text{H.c.}$ From the $\mathcal{L}_{\text{Yukawa}}$, we obtain

$$
\begin{aligned}
m_U &= -\frac{1}{\sqrt{2}}h^U\omega, \quad [m_D]_{\alpha\beta} = -\frac{1}{\sqrt{2}}h^D_{\alpha\beta}\omega, \\
[m_u]_{\alpha a} &= \frac{1}{\sqrt{2}}h^u_{\alpha a}v, \quad [m_u]_{3a} = -\frac{1}{\sqrt{2}}h^u_a u, \\
[m_d]_{\alpha a} &= -\frac{1}{\sqrt{2}}h^d_{\alpha a}v, \quad [m_d]_{3a} = -\frac{1}{\sqrt{2}}h^d_a v, \\
[m_e]_{ab} &= -\frac{1}{\sqrt{2}}h^e_{ab}v, \quad [m^D_\nu]_{ab} = -\frac{1}{\sqrt{2}}h^\nu_{ab}u.
\end{aligned}
$$

The right handed neutrinos get Majorana masses in the form $-\frac{1}{2}\bar{\nu}^c_R m^M_\nu \nu_R + \text{H.c.}$, where

$$[m^M_\nu]_{ab} = -\sqrt{2}h'^\nu_{ab}\Lambda.$$

The observed neutrinos ($\sim \nu_L$) naturally get small masses via a type I seesaw mechanism,

$$m^{\text{eff}}_\nu = -m^D_\nu (m^M_\nu)^{-1}(m^D_\nu)^T \sim \frac{(h^\nu)^2}{h'^\nu}\frac{u^2}{\Lambda}.$$

The masses of N_R can be generated via an effective operator invariant under the 3-3-1-1 symmetry

$$\frac{\lambda_{ab}}{M}\bar{\psi}^c_{aL}\psi_{bL}(\chi\chi)^* + \text{H.c},$$

$$[m_{N_R}]_{ab} = -\lambda_{ab}\frac{\omega^2}{M}.$$

Assume that $M \sim \omega$ then $m_{N_R} \sim \omega$

6. Summary

▷ After spontaneous symmetry breaking, there are

- 9 goldstone bosons $A_4, G_Z, G_{Z'}, G_X, G_X^*, G_Y^\pm, G_W^\pm$,
- 9 massive gauge bosons $Z^N, Z^1, Z^2, X^0, X^{0*}, Y^\pm, W^\pm$, and one massless γ,
- 4 neutral Higgs bosons H, H_1, H_2, H_3, one massive pseudoscalar A, complex Higgs H', H'^*, 4 charged scalars $H_4^\pm, H_5^\pm$

▷ H_3, Z^N, ν_R have mass in order $\mathcal{O}(\Lambda)$.
All other new particles, $A, H_1, H_2, H_4^\pm, H_5^\pm, H', H'^*$, $Z_\mu^2, X_\mu^0, X_\mu^{0*}, Y_\mu^\pm$, U, D_α, N_R, have mass in order $\mathcal{O}(\omega)$.

▷ In this model, $L(G_X, H'^*, H_4^-, G_Y^-, X^0, Y^-) = 1$ while the remaining Higgs and gauge bosons have zero lepton number.

▷ The Majorana masses of the right handed neutrinos violate L with ± 2 units $\rightarrow$ The decay of Majorana RH neutrino can generate the lepton asymmetry.

▷ $P(N_R, X, Y, U, D, H_4, H') = -1$ and all other particles have $P = +1$. The lightest and neutral particle among odd parity particles can be a dark matter candidate.

Thank you